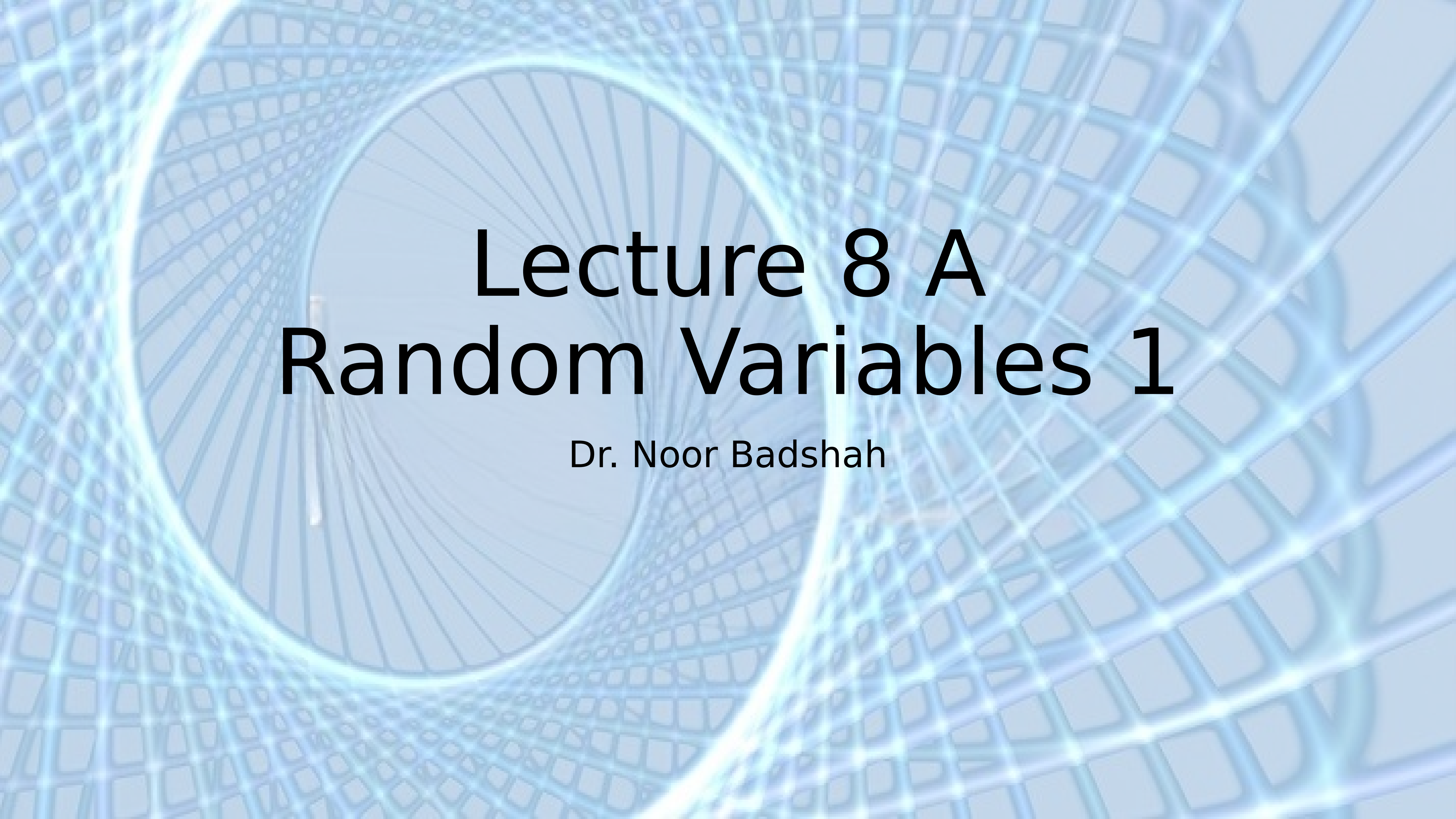


Lecture 8 A
Random Variables 1

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Radom Variable

- Those variables whose values are determined by outcomes of an experiment.
- For example: A coin is tossed and represent no of heads appeared.

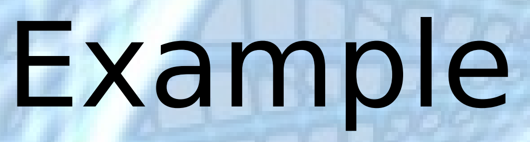
- Two coins are tossed and represents no heads

- If two dices are thrown and represents sum of the dots shown:

RV Discrete RV and Continuous RVs

Probability Distribution (PD) (Discrete RVs)

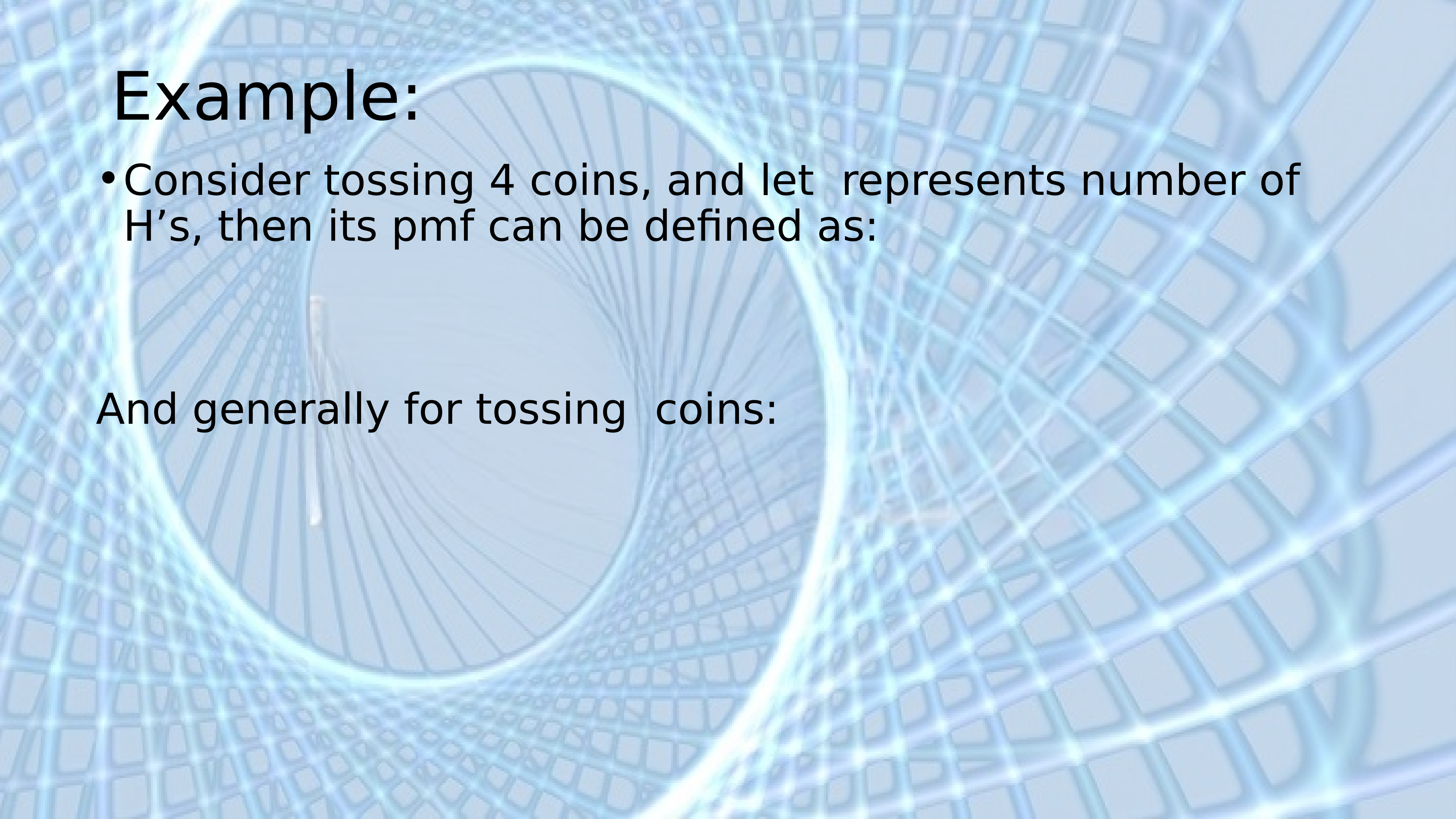
- RV Values Outcomes Probability
- PD also known as probability mass function and is a table which contains values of RV and their corresponding probabilities:
- A coin is tossed and is the number of heads:



Example

- Consider a random experiment of tossing 2 coins, and let represents no. of H’s, then its pmf will be:

- Consider a random experiment of tossing 3 coins, and let represents no. of H’s, then its pmf will be:



Example:

- Consider tossing 4 coins, and let represents number of H’s, then its pmf can be defined as:

And generally for tossing coins:

Is Every table of values is PD?

- No.
- Conditions: 1.   2.
- If the given table represents a PD, c=?

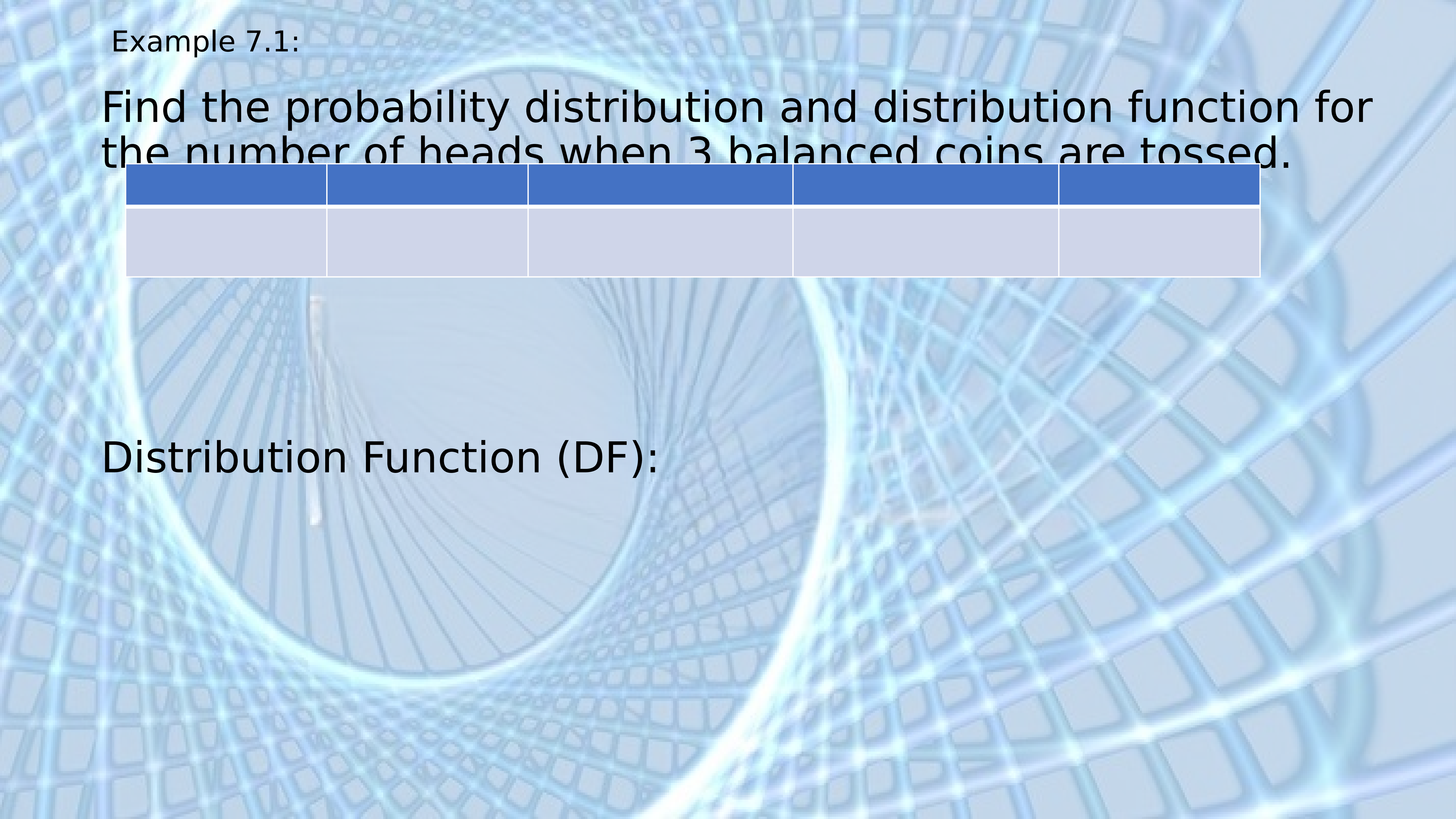
	1	2	3	4
)	0.1	0.4	0.2	c

-

Distribution Function (Cumulative Probability)

- Representation Sum of probabilities less than or equal to
- Example: From previous slide:
- will represent DF if

1. 2. 3. is a non decreasing function. 4. should be continuous at least from right.



Example 7.1:

Find the probability distribution and distribution function for the number of heads when 3 balanced coins are tossed.

Distribution Function (DF):

Examples 7.2 (a) Find the probability distribution of the sum of the dots when two fair dice are [illegible]

(b) Use the probability distribution to find the probabilities of obtaining (i) a sum of 8 or 11, [illegible] that is greater than 8, (iii) a sum that is greater than 5 but less than or equal to 10.

Let X be the random variable representing the sum of dots which appear on the dice. The[illegible] values of the r.v. are 2, 3, 4, …, 12. The probabilities of these values are computed as below;

Therefore the desired probability distribution of the r.v. X is

x_i	2	3	4	5	6	7	8	9	10	11	12
$f(x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

It is interesting to note that this result may also be expressed by the equation as

$$f(x) = \frac{6-|7-x|}{36}, \quad \text{for } x = 2, 3, 4, \ldots, 12.$$

(b) Using the probability distribution, we get the required probabilities as follows:

i) $P(\text{a sum of 8 or 11}) = P[(X=8) \text{ or } (X=11)]$

$= P(X=8) + P(X=11)$

$= f(8) + f(11) = \frac{5}{36} + \frac{2}{36} = \frac{7}{36}$

ii) $P(\text{a sum that is greater than 8})$

$= P(X > 8)$

$= P(X=9) + P(X=10) + P(X=11) + P(X=12)$

$= f(9) + f(10) + f(11) + f(12)$

$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36}$

iii) $P(\text{a sum that is greater than 5 but less than or equal to 10})$

$= P(5 < X \leq 10)$

$= P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= f(6) + f(7) + f(8) + f(9) + f(10)$

$= \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} = \frac{23}{36}$

Cont RVs

- Probability Density Function (pdf):

A function will be a pdf for a continuous random variable if

Note: If is a continuous RV then

and

- Distribution Function:
- Note that

Example 7.3 (a) Find the value of k so that the function $f(x)$ defined as follows, may be a [illegible]
function

$$f(x) = kx, \quad 0 \leq x \leq 2$$
$$= 0, \quad \text{elsewhere}$$

(b) Find also the probability that both of two sample values will exceed 1.

(c) Compute the distribution function $F(x)$.

a) The function f(x) will be a density function, if

i) $f(x) \geq 0$ for every x, and

ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

The first condition is satisfied when $k \geq 0$. The second condition will be satisfied, if $\int_{-\infty}^{\infty} f$[illegible]

i.e. if $$1 = \int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx$$

i.e. if $1 = 0 + \left[k\frac{x^2}{2}\right]_0^2 + 0 = 2k$

This gives $k = \frac{1}{2}$

Hence $f(x) = \begin{cases} \frac{x}{2}, & \text{for } 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$

$P(X>1)$ = areas of shaded region

$$= \int_1^2 f(x)\,dx$$

$$= \int_1^2 \frac{x}{2}\,dx = \left[\frac{x^2}{4}\right]_1^2 = \frac{3}{4}$$

$\therefore$ P(two sample values exceeding one) $= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

To compute the distribution function, we find

$$F(x) = P(X < x) = \int_{-\infty}^{x} f(x)\,dx$$

[illegible] x such that $-\infty < x \le 0, F(x) = \int_{-\infty}^{x} 0\,dx = 0,$

[illegible] ≤ 2, we have $F(x) = \int_{-\infty}^{0} 0\,dx + \int_0^x \left(\frac{x}{2}\right)dx = \left[\frac{x^2}{4}\right]_0^x = \frac{x^2}{4},$

[illegible]y for $x > 2$, we have $F(x) = \int_{-\infty}^{0} 0\,dx + \int_0^2 \frac{x}{2}\,dx + \int_2^{\infty} 0\,dx = 1$

[illegible] $F(x) = 0$, $\quad$ *for* $x < 0$

$= \frac{x^2}{4}$, $\quad$ *for* $0 \le x \le 2$

$= 1$, $\quad$ *for* $x > 2$

Example 7.4 A r.v. X is of continuous type with p.d.f.

$$f(x) = 2x, \quad 0 < x < 1,$$
$$= 0, \quad \text{elsewhere.}$$

Find (i) $P\left(X = \frac{1}{2}\right)$, (ii) $P\left(X \le \frac{1}{2}\right)$, (iii) $P\left(X > \frac{1}{4}\right)$, iv) $P\left(\frac{1}{4} \le X < \frac{1}{2}\right)$

(v) $P\left(X \le \frac{1}{2} \mid \frac{1}{3} \le X \le \frac{2}{3}\right)$.

Clearly $f(x) \ge 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 2x\,dx = 1.$

i) Since $f(x)$ is a continuous probability function, therefore

$$P\left(X = \frac{1}{2}\right) = 0.$$

ii) $P\left(X \le \frac{1}{2}\right) = \int_{-\infty}^{0} 0\,dx + \int_0^{1/2} 2x\,dx = 0 + [x^2]_0^{1/2} = \frac{1}{4}$

iii) $P\left(X > \frac{1}{4}\right) = \int_{1/4}^{1} 2x\,dx + \int_1^{\infty} 0\,dx = [x^2]_{1/4}^{1} + 0 = \frac{15}{16}$

iv) $P\left(\frac{1}{4} \le X < \frac{1}{2}\right) = \int_{1/4}^{1/2} 2x\,dx = [x^2]_{1/4}^{1/2} = \frac{3}{16}$

v) Applying the definition of conditional probability, we get

$$P\left(X \le \frac{1}{2} \mid \frac{1}{3} \le X \le \frac{2}{3}\right) = \frac{P\left(\frac{1}{3} \le X \le \frac{1}{2}\right)}{P\left(\frac{1}{3} \le X \le \frac{2}{3}\right)} = \frac{\int_{1/3}^{1/2} 2x\,dx}{\int_{1/3}^{2/3} 2x\,dx}$$

$$= [x^2]_{1/3}^{1/2} \div [x^2]_{1/3}^{2/3}$$

$$= \frac{5}{36} \times \frac{9}{3} = \frac{5}{12}.$$

Example 7.5 A continuous r.v. X has the *d.f.* $F(x)$ as follows:

$$F(X) = 0, \qquad \text{for } x < 0,$$

$$= \frac{2x^2}{5}, \qquad \text{for } 0 < x \leq 1,$$

$$= -\frac{3}{5} + \frac{2}{5}\left(3x - \frac{x^2}{2}\right), \qquad \text{for } 1 < x \leq 2,$$

$$= 1 \qquad \text{for } x > 2.$$

Find the p.d. and $P(|X| < 1.5)$.

By definition, we have $f(x) = \frac{d}{dx}F(x)$.

Therefore
$$f(x) = \frac{4x}{5} \quad \text{for } 0 < x \leq 1$$
$$= \frac{2}{5}(3-x) \quad \text{for } 1 < x \leq 2$$
$$= 0 \quad \text{elsewhere.}$$

$$P(|X| < 1.5) = P(-1.5 < X < 1.5)$$

$$= \int_{-\infty}^{-1.5} 0\, dx + \int_{-1.5}^{0} 0\, dx + \int_{0}^{1} \frac{4x}{5} dx + \int_{1}^{1.5} \frac{2(3-x)}{5} dx$$

$$= 0 + 0 + \left[\frac{2x^2}{5}\right]_0^1 + \left[\frac{2}{5}\left(3x - \frac{x^2}{2}\right)\right]_1^{1.5}$$

$$= \frac{2}{5} + \frac{2}{5}\left[\left(4.5 - \frac{2.25}{2}\right) - \left(3 - \frac{1}{2}\right)\right]$$

$$= 0.40 + 0.35 = 0.75.$$